

Towards Realistic Evolution of Atmospheric Internal Gravity Wave Spectra: Coupled Modulation and Wave-Wave Interactions

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Motivation

- Gravity-wave parameterization:** Atmospheric internal gravity waves (IGWs) transport energy and momentum across scales, influencing major atmospheric circulation processes such as the **quasi-biennial oscillation (QBO)**, and **mesospheric circulation** etc. However, many IGW scales remain unresolved in weather and climate models, such as **ICON**, requiring parameterization.
- Towards transient parameterizations:** Conventional gravity-wave parameterizations often assume steady-state propagation. Modern frameworks such as **MS-GWaM**, implemented in **ICON**, use Lagrangian WKB ray tracing to describe transient propagation, background-flow modulation, and wave-mean-flow interactions.
- Missing nonlinear interactions:** Wave-wave interactions redistribute spectral energy and generate new wave components, but are generally neglected in existing gravity-wave parameterizations.
- Gap between existing theories:** WKB approaches capture transient modulation, while **weak wave turbulence (WWT)** theory describes nonlinear spectral transfer, typically under homogeneous and stationary background conditions. Their combined effects remain insufficiently understood.
- Why coupling matters:** Background-flow modulation changes wavevectors and intrinsic frequencies, thereby influencing triad resonance and nonlinear energy transfer. The resulting spectral redistribution may also alter wave momentum fluxes and **mean-flow forcing**.

Research Objective

To model and investigate the coupled effects of transient WKB modulation and resonant wave-wave interactions on the spatiotemporal evolution of atmospheric IGW spectra and their feedback on the resolved mean flow.

Modal Validation

Benchmark: Coupled modulation and wave-wave interactions (WKB-WW) simulations using **MS-GWaM** are compared with high-resolution wave-resolving simulations (WRS) using **PinCFLOW**.

Interaction regimes: Elastic scattering (ES), parametric subharmonic instability (PSI), and induced diffusion (ID).

Simulation setup: A 2D domain (50×40 km) with constant stratification ($N = 0.01s^{-1}$) and initial sinusoidal wind, $\tilde{u}(z,0) = u_0 \sin(2\pi z/H)$, $H = 40$ km. Boussinesq atmospheric. Alternative shear profiles show similar agreement (not shown).

Sensitivity: PSI simulations with varying background shear strength and initial wave amplitude.

Validation Results:

- The model captures modal-energy transfers and the growth of generated wave modes for the various shear strengths and amplitudes.
- Agreement with WRS is strongest at weak amplitudes, with larger deviations in some strongly nonlinear cases (due to failure of the weak amplitude condition).

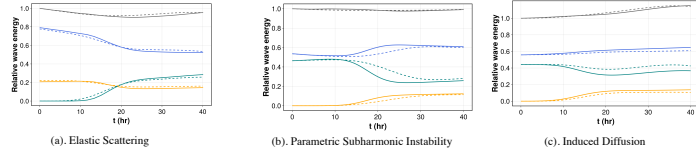


Figure 2. Validation against wave-resolving simulations. Time evolution of normalized modal wave energies for (a) ES, (b) PSI, and (c) ID interactions. Final modal-energy distributions for PSI under varying (d) background-wind amplitude and (e) initial wave amplitude. Solid curves denote coupled WKB-WW simulations, while dashed curves denote high-resolution WRS results.

Spectral Evolution and Main Results

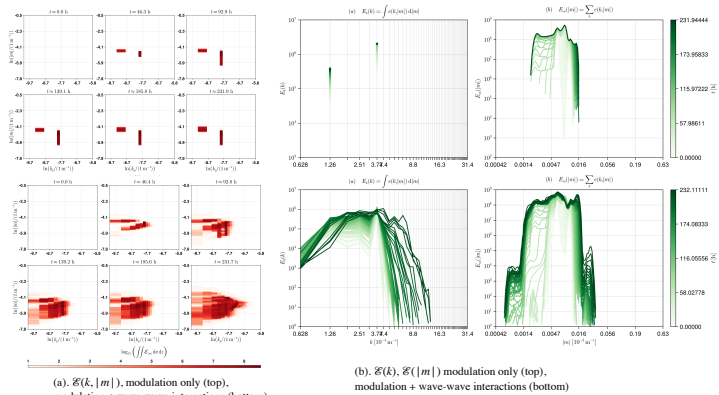


Figure 4. Long-time evolution of internal gravity-wave spectra and induced mean flow. Comparison of WKB-WW and modulation-only simulations using (a) spatially integrated energy distributions in (k, m) -space, (b) m integrated and k integrated energy spectra, and (c) vertical profiles of the wave-induced mean wind at selected times

Conclusions

- A unified **MS-GWaM-based** framework couples transient **WKB modulation** with **resonant wave-wave interactions** and **two way wave-mean-flow feedback**.
- Compared with modulation-only simulations, wave-wave interactions **generate new spectral components**, **redistribute energy across wavenumbers**, and **modify the wave-induced mean-flow response**.

Outlook

- Investigate the emergence of spectral scaling laws and the long-time evolution of IGW spectra under coupled modulation and nonlinear interactions.
- Extend the framework towards **3D rotating**, and **compressible atmospheric configurations**, with the long-term goal of improving gravity-wave parameterizations in weather and climate models.

Modelling and Numerical Implementation

Coupled Modulation-Triad Framework for Transient Internal Gravity Waves

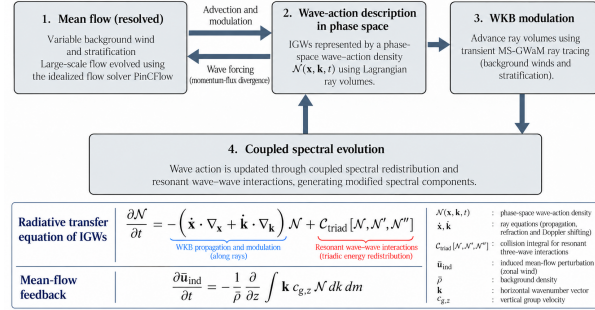


Figure 1. The coupled framework combines transient WKB modulation with local resonant wave-wave interactions, allowing the wave spectrum and its feedback on the resolved mean flow to evolve together.

Wave-Wave Interactions and Mean-Flow Response

Simulation setup: Elastic scattering (ES) under vertically varying stratification, with no initially imposed background shear.

Comparison: The wave-induced mean wind is examined using high-resolution wave-resolving simulations (WRS), coupled WKB-WW simulations, and WKB modulation only simulations.

Physical Implications:

- WKB-WW captures the principal mean-flow structure seen in WRS, including an additional contribution associated with the generated wave mode that is absent from the modulation-only model.
- The secondary mode induces only weak mean-flow changes in the present Boussinesq simulations. However, density-driven wave-amplitude growth with altitude may produce stronger momentum-flux convergence and mean-flow forcing in compressible or pseudo-incompressible atmospheres.

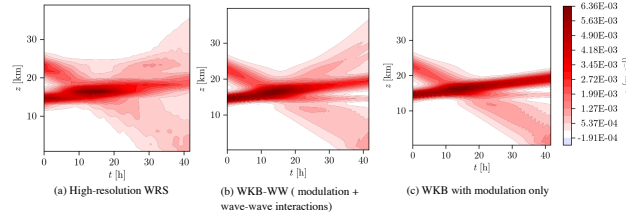


Figure 3. Evolution of wave-induced mean wind: Time-height distribution of the wave-induced horizontal mean wind ($u_{ind}(z, t)$) obtained from (a) coarse-grained WRS, (b) WKB-WW, and (c) WKB modulation only, for the ES configuration under vertically varying stratification.

Simulation setup: Long-time simulations initialized continuously forced with two narrow spectral bands in a vertically varying background wind and stratification.

Comparison: WKB modulation alone versus coupled WKB-WW dynamics, examining two-dimensional energy distributions, integrated spectra, and wave-induced mean winds.

Main Observations:

- WKB modulation only:** Energy remains concentrated at the initial horizontal wavenumbers, while vertical wavenumbers evolve through background-flow modulation.
- Coupled WKB-WW:** Nonlinear interactions generate new spectral components and redistribute energy across both horizontal and vertical wavenumbers.
- Mean-flow response:** The resulting spectral redistribution also modifies the wave-induced mean wind, with increasingly noticeable differences at later times.

Acknowledgements

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