



Nonlinearity in Madden-Julian Oscillation Propagation: ENSO Modulation and a Data-Driven Dynamical Perspective



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1. INTRODUCTION & BACKGROUND

The Madden–Julian Oscillation (MJO): The primary mode of tropical intraseasonal variability (30–90 day period), characterized by a planet-scale convective complex propagating eastward at ~5 m/s across the equatorial Indian and Pacific Oceans.

Multi-Scale Organization & Global Impact: Initiating as deep convective pulses over the equatorial Indian Ocean, the MJO couples with planet-scale atmospheric circulation, strongly modulating Asian-Australian-South American monsoons, tropical cyclogenesis, Atlantic trade winds, and global atmospheric teleconnections (Zhang, 2005; Jiang et al., 2020).

Current NWP/GCM Forecasting Challenges: State-of-the-art global circulation models (GCMs) and numerical weather prediction systems present persistent biases in capturing MJO propagation, amplitude modulation, and its interactions with low-frequency climate modes (Majda & Stechmann, 2011; Zhang et al., 2020).

Research Objective: Characterize the intrinsic nonlinear dynamics of MJO amplitude and phase progression, and quantify how **El Niño–Southern Oscillation (ENSO)** background climate phases (El Niño, La Niña, and Neutral) modulate its governing differential equations using a data-driven dynamical systems approach.

2. MATERIALS AND METHODS

Data & Stratification: Daily multivariate RMM1 and RMM2 index series (Wheeler & Hendon, 2004) filtered for active MJO events (amplitude $A > 1$). Stratified into El Niño (EN), La Niña (LN), and Neutral (N) phases, evaluated with annual cycle removal (crca) and without annual cycle removal (srca).

PySINDy Machine-Learning Framework: Sparse Identification of Nonlinear Dynamics (SINDy) extracts interpretable ordinary differential equations $\text{dx}/\text{dt} = \mathbf{f}(\mathbf{x})$ directly from observational data (Brunton et al., 2016; de Silva et al., 2020; Kaptanoglu et al., 2021). It evaluates a high-dimensional functional candidate library $\mathbf{\Theta}(\mathbf{X})$ and isolates sparse governing physical interactions via Sequentially Thresholded Least Squares (STLSQ) regression.

Ensemble Modeling Pipeline: State vector $\mathbf{X} = [\text{Amplitude (A)}, \text{Phase (F)}]$. Functional library: 5th-degree polynomial library (`PolynomialLibrary(degree=5)`) encompassing all non-linear interaction terms between A and F. Ensemble Optimizer (Bagging): 1,000 independent bootstrap models trained on 50% random sub-samples with STLSQ threshold = 0.001.

3. RESULTS: INFERRED DIFFERENTIAL EQUATIONS

The ensemble PySINDy regression extracted sparse, interpretable differential equations governing MJO Amplitude (A) and Phase (F) progression across the 5 experimental scenarios:

Scenario / Regime	Amplitude Equation (dA/dt)	Phase Equation (dF/dt)
1. EN_{crca} (El Niño, crca)	$\text{dA}/\text{dt} = 0.297 \text{ A} - 0.519 \text{ A}^2 + 0.344 \text{ A}^3 - 0.101 \text{ A}^4 + 0.010 \text{ A}^5$	$\text{dF}/\text{dt} = 1.816 \text{ A} - 2.582 \text{ A}^2 + 1.131 \text{ A}^3 - 0.136 \text{ A}^4 - 0.020 \text{ A}^5$
2. LN_{crca} (La Niña, crca)	$\text{dA}/\text{dt} = 0.100 \text{ A} - 0.191 \text{ A}^2 + 0.182 \text{ A}^3 - 0.069 \text{ A}^4 + 0.010 \text{ A}^5$	$\text{dF}/\text{dt} = 2.999 \text{ A} - 5.508 \text{ A}^2 + 3.899 \text{ A}^3 - 0.747 \text{ A}^4 + 0.033 \text{ A}^5$
3. N_{crca} (Neutral, crca)	$\text{dA}/\text{dt} = 0.101 \text{ A} - 0.225 \text{ A}^2 + 0.185 \text{ A}^3 - 0.049 \text{ A}^4 + 0.003 \text{ A}^5$	$\text{dF}/\text{dt} = 6.208 \text{ A} + 0.001 \text{ F} - 13.257 \text{ A}^2 - 0.002 \text{ AF} + 10.405 \text{ A}^3 + 0.002 \text{ A}^2\text{F} - 3.690 \text{ A}^4 - 0.001 \text{ A}^3\text{F} + 0.484 \text{ A}^5$
4. Full_{srca} (No Removal)	$\text{dA}/\text{dt} = -0.001 \text{ A}^2$	$\text{dF}/\text{dt} = 0.229 \text{ A} - 0.199 \text{ A}^2 + 0.089 \text{ A}^3 - 0.019 \text{ A}^4 + 0.002 \text{ A}^5$
5. Full_{crca} (Full Series, crca)	$\text{dA}/\text{dt} = -0.010 \text{ F} - 0.012 \text{ A}^2 - 0.004 \text{ AF} + 0.009 \text{ A}^2\text{F} + 0.002 \text{ F}^3 + 0.014 \text{ A}^4 + 0.015 \text{ A}^3\text{F} + 0.002 \text{ A}^2\text{F}^2 - 0.007 \text{ A}^5 - 0.009 \text{ A}^4\text{F} - 0.001 \text{ A}^2\text{F}^3$	$\text{dF}/\text{dt} = -0.010 \text{ A} - 0.003 \text{ F} - 0.036 \text{ A}^2 + 0.006 \text{ AF} + 0.173 \text{ F}^2 + 0.041 \text{ A}^3 + 0.045 \text{ A}^2\text{F} + 0.033 \text{ AF}^2 - 0.027 \text{ A}^4 - 0.038 \text{ A}^3\text{F} + 0.029 \text{ A}^2\text{F}^2 + 0.001 \text{ AF}^3 - 0.030 \text{ F}^4 + 0.008 \text{ A}^5 + 0.011 \text{ A}^4\text{F} - 0.013 \text{ A}^3\text{F}^2 - 0.005 \text{ A}^2\text{F}^3 - 0.006 \text{ AF}^4$

4. DETAILED DISCUSSION & NONLINEAR DYNAMICS

Unidirectional Amplitude-to-Phase coupling EN & LN Regimes)

- In active ENSO phases (**EN_{crca}** & **LN_{crca}**), phase rate velocity (dF/dt) is strictly driven by higher-order polynomial powers of amplitude (A^n).
- Amplitude evolution (dA/dt) remains autonomous and uncoupled from phase (F), demonstrating driven phase propagation dynamics without direct feedback.

Enhanced Sensitivity & Instability in La Niña (LN vs. EN)

- Phase propagation coefficients in La Niña (**LN_{crca}**) are substantially larger than in El Niño (**EN_{crca}**) – e.g., primary linear term coefficient is **2.999 A** in LN vs. **1.816 A** in EN.
- This indicates heightened phase velocity sensitivity to MJO amplitude fluctuations during La Niña, signaling a more dynamic and unstable propagation regime.

State-Space Topology Shift in Neutral ENSO Phase

- Neutral ENSO (**N_{crca}**) introduces explicit phase dependence (0.001 F) and mixed interaction terms (AF , A^2F , A^3F) alongside a dominant linear term (**6.208 A**).
- This reveals that large-scale boundary conditions fundamentally restructure the system's phase-space topology and coupling mechanisms.

Weakly Nonlinear Cross-Coupling in Full Unstratified Record

- Analyzing the full record without ENSO stratification (**Full_{crca}**) unmask bidirectional cross-couplings between amplitude and phase with smaller magnitude coefficients.
- This represents a weakly nonlinear regime where phase feedbacks subtly modify amplitude evolution.

Masking Effect of Low-Frequency Seasonal Signal

- Retaining the annual cycle (**Full_{srca}**) causes the dominant seasonal signal to obscure intraseasonal nonlinearities, reducing dA/dt to a trivial term (-0.001 A^2).
- This proves that annual cycle removal is strictly mandatory for isolating true intraseasonal nonlinear dynamics.

5. FINAL CONSIDERATIONS & FUTURE WORK

Key Takeaways & Conclusions:

- Data-driven sparse regression via **PySINDy** successfully recovers interpretable, low-dimensional ODEs for MJO dynamics.
- ENSO climate regimes alter not only MJO mean amplitude, but fundamentally transform its underlying mathematical coupling structure.
- Filtering the annual cycle is crucial to prevent low-frequency signals from masking intraseasonal nonlinearities.

Future Research Directions:

- Phase-Space Reconstruction:** Reconstruct state space using Time-Delay Embedding (Takens' Theorem) to examine attractor geometry and invariants (Alligood et al., 1998; Kantz & Schreiber, 2004).
- External Climate Forcings (IOD & QBO):** Incorporate external climate drivers—specifically the **Indian Ocean Dipole (IOD)** and the **Quasi-Biennial Oscillation (QBO)**—as explicit forcing terms in future PySINDy models.

6. REFERENCES

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